PANM 15

Consistent nonlocal tangent operator for damage-plasticity model

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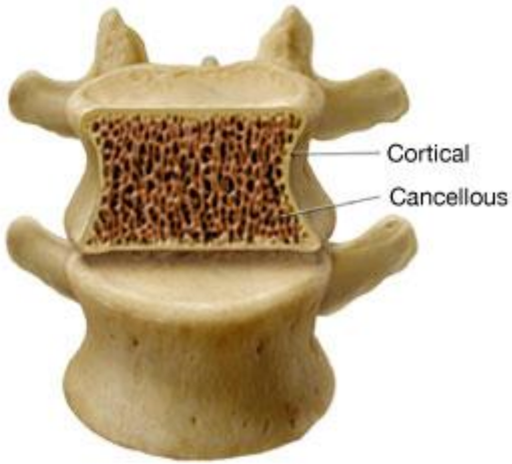
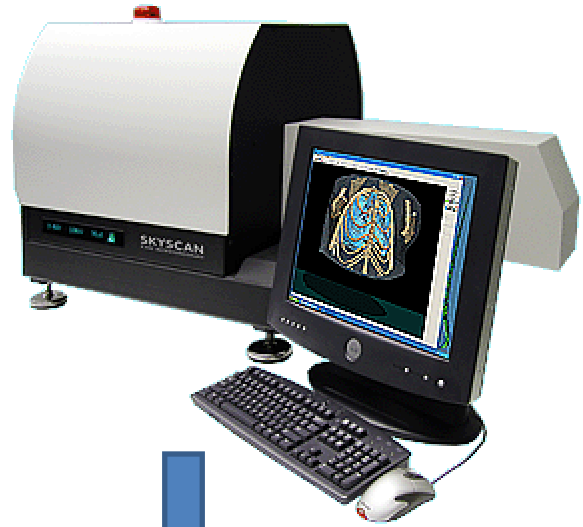
Obsah

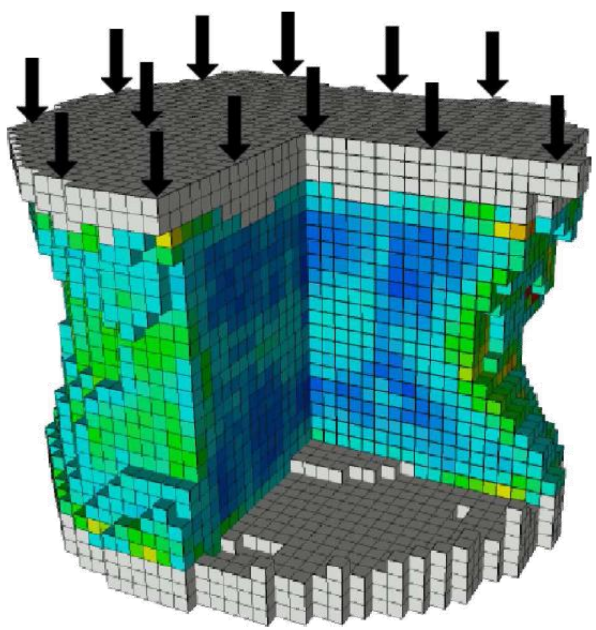
- Motivace
- Model spongiózní kosti
  - Lokální
  - Nelokální

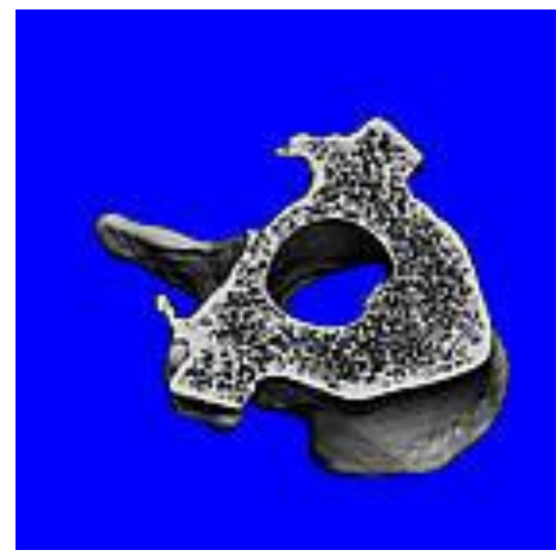
- Závěr

Motivace



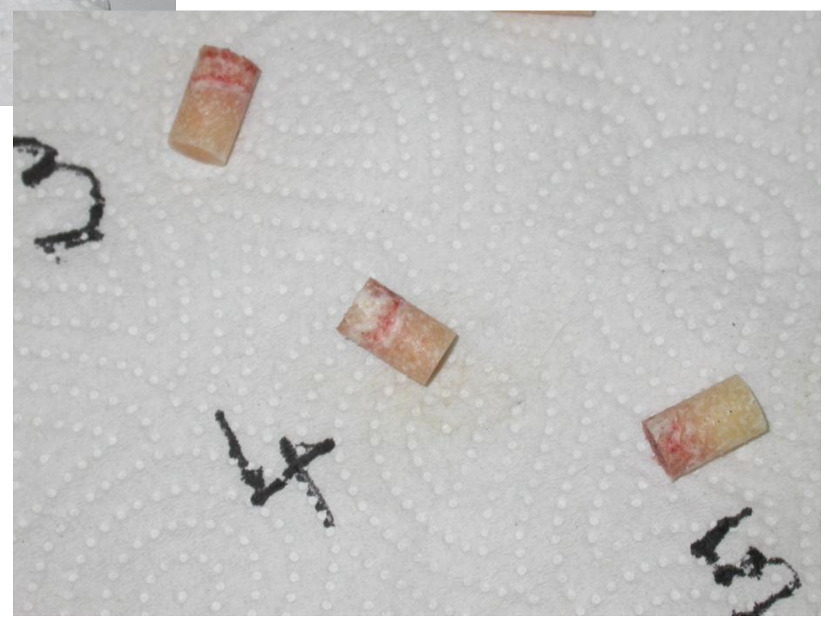




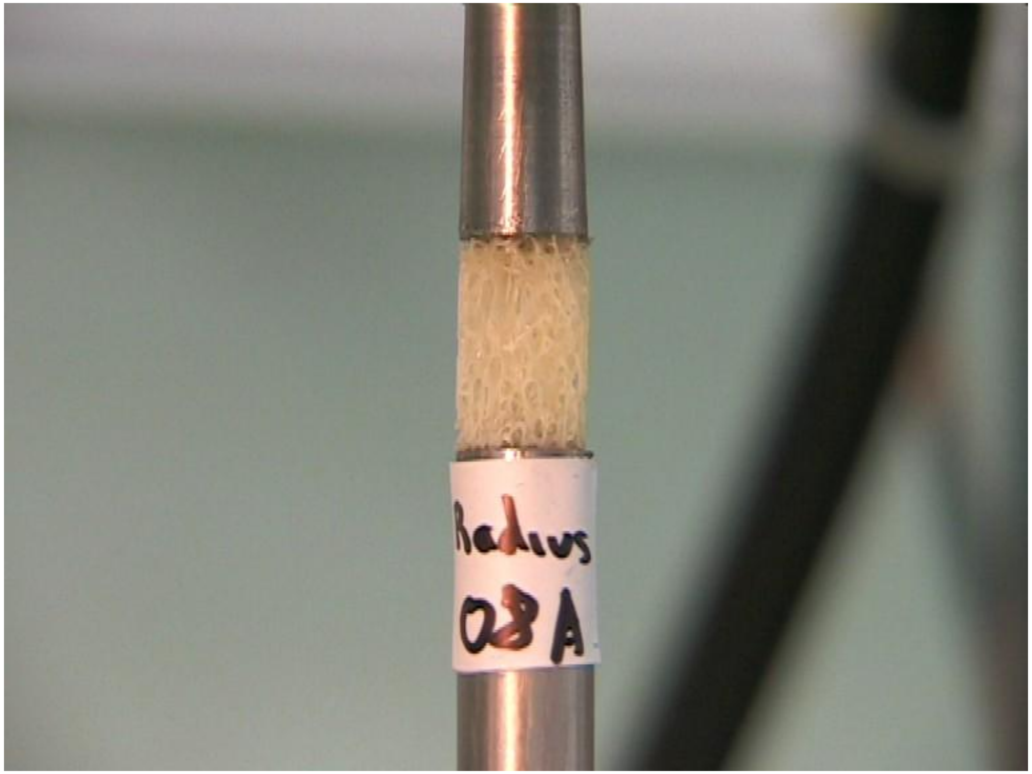


Motivace

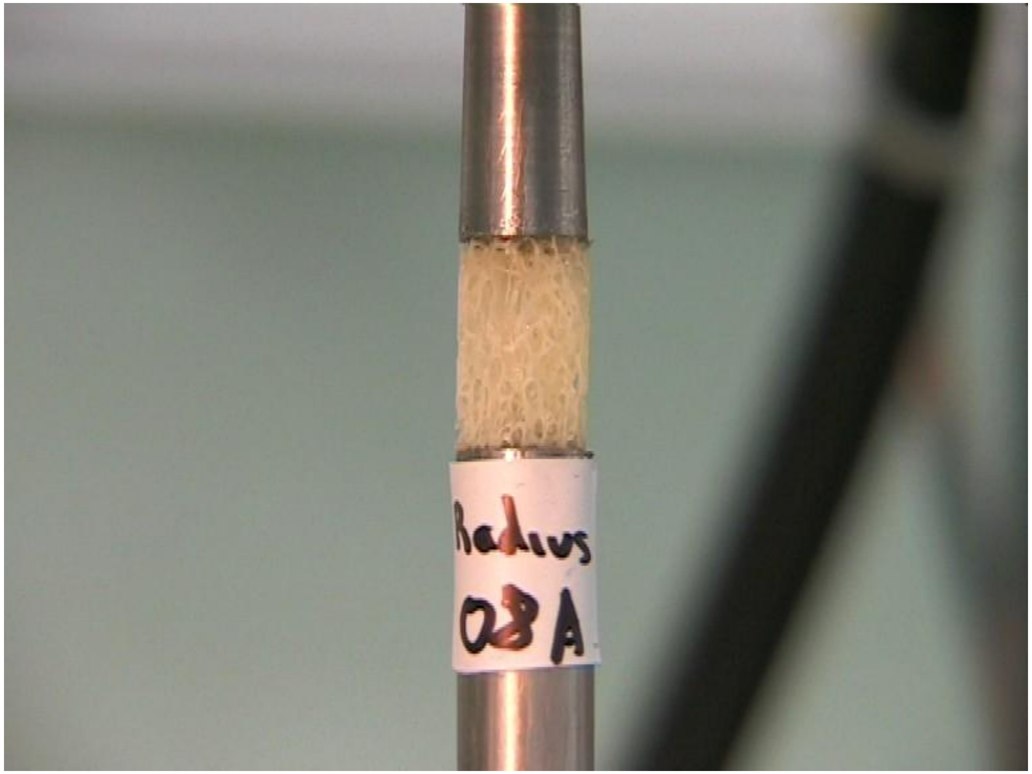
Motivace



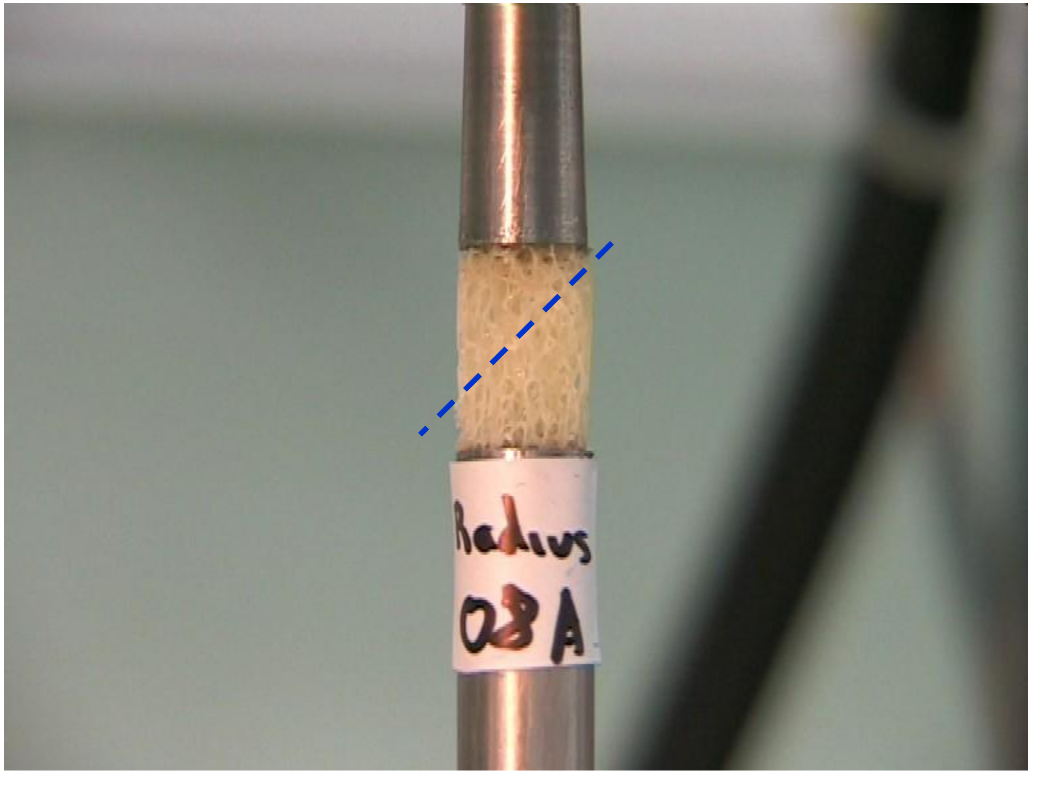
Motivace



Motivace



Motivace



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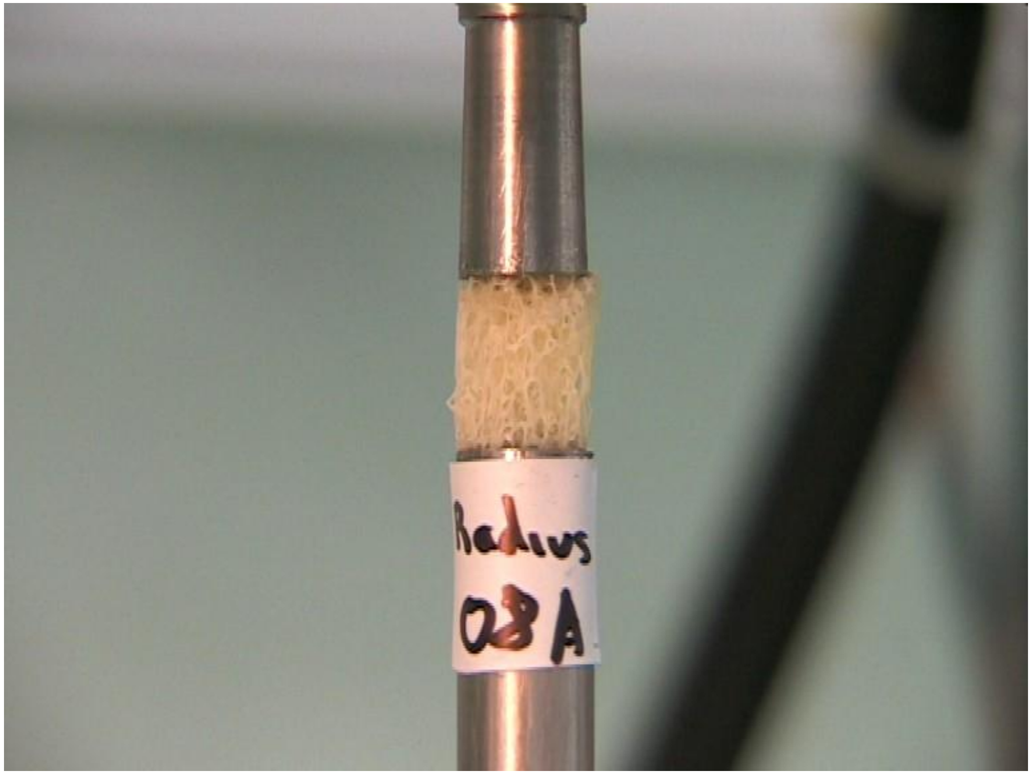
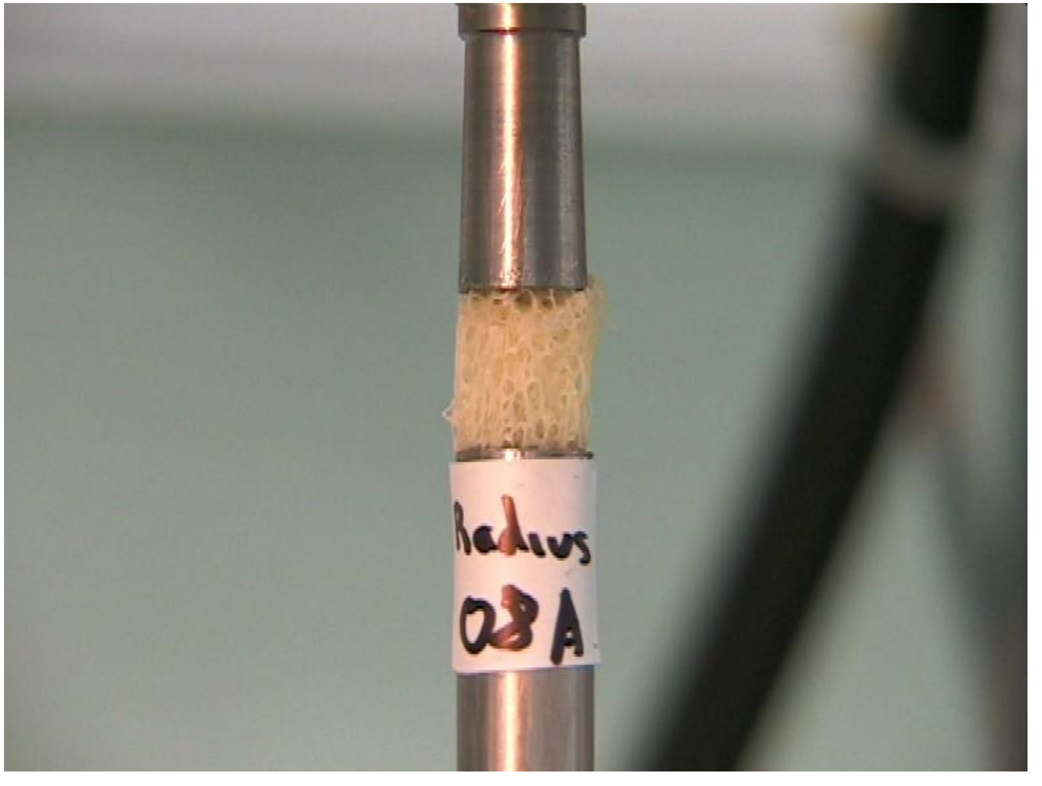
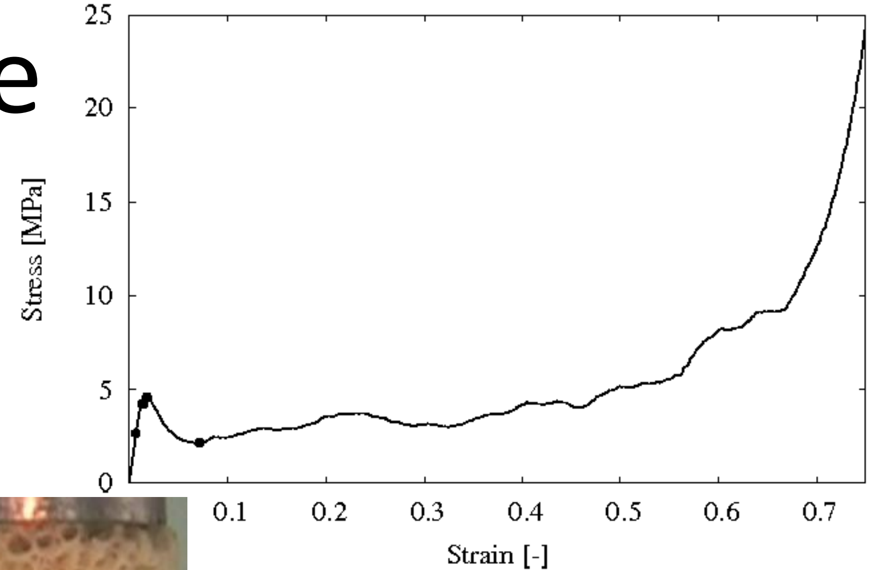
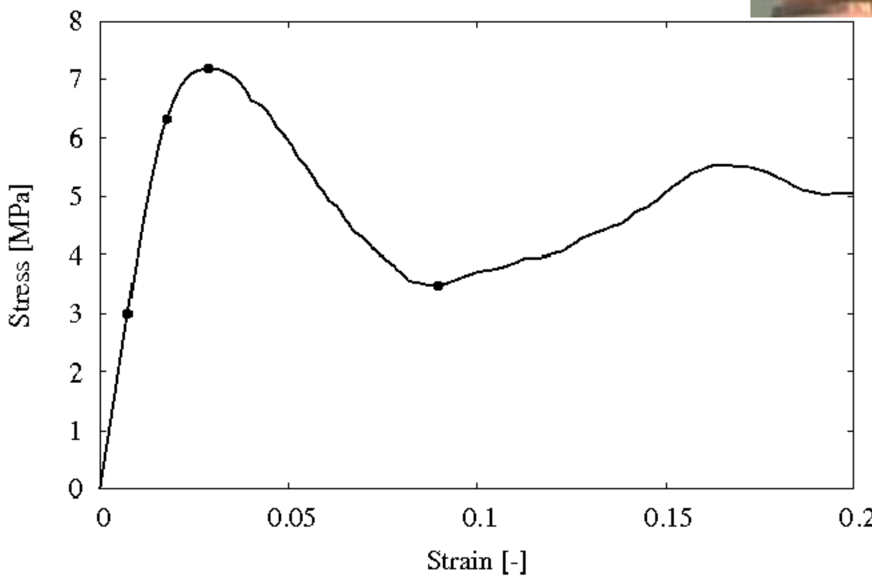


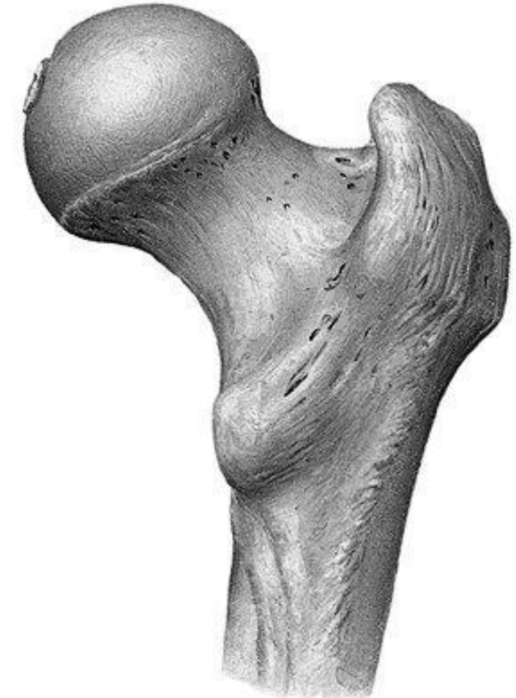
Motivace

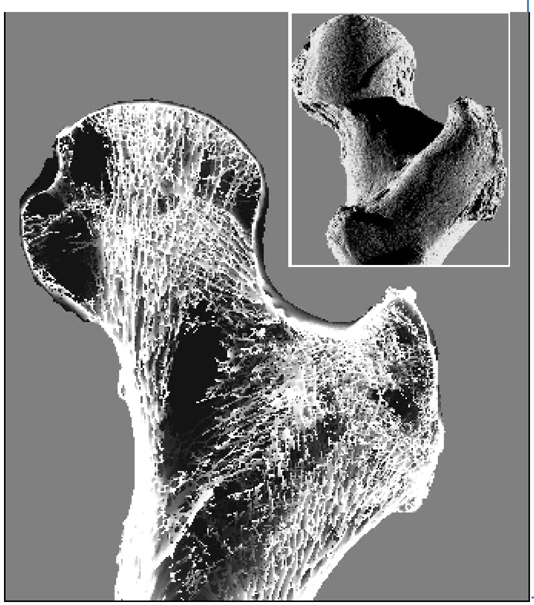


Motivace


Stress [MPa]
Strain [-]
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Stress [MPa]
Strain [-]
0
1
2
3
4
5
6
7
8
0
0.05
0.1
0.15
0.2





Základní rovnice

$$\nabla \cdot \Big((1-\omega)\mathbf{D}_e : \underbrace{(\nabla^s \mathbf{u} - \boldsymbol{\varepsilon}^p)}_{\boldsymbol{\varepsilon}_e} \Big) + \mathbf{X} = 0$$

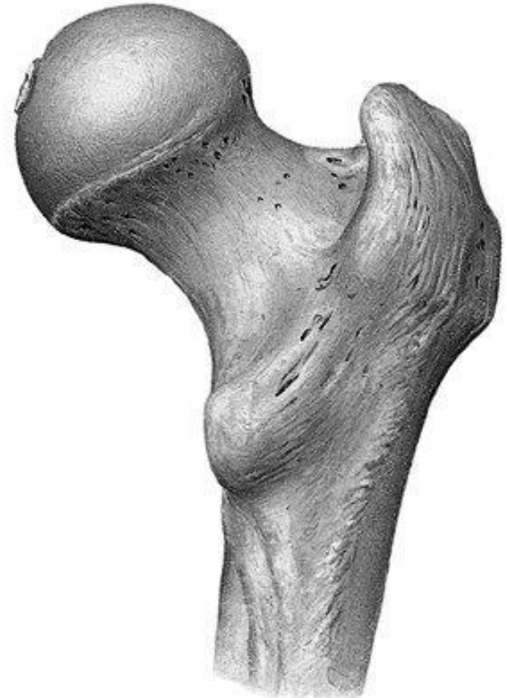
$$\boldsymbol{\varepsilon} = \nabla^s \mathbf{u} \qquad \mathbf{u}\big|_{\Gamma^u} = \mathbf{u}^*$$

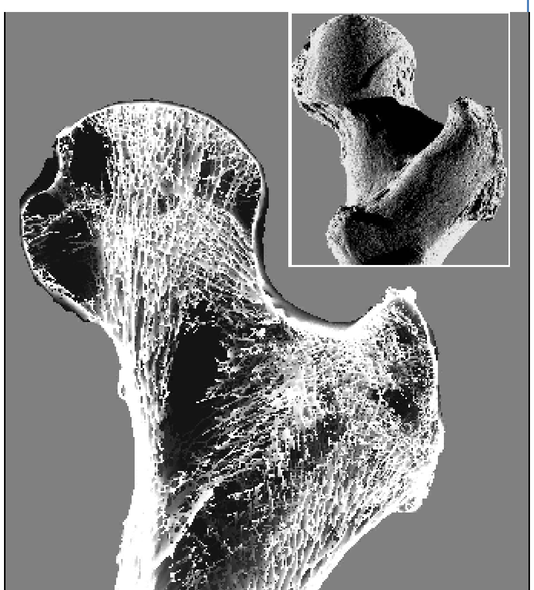
$$\bar{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{(1-\omega)}$$

$$E_\sigma := \left\{ (\bar{\boldsymbol{\sigma}}, \kappa) \in S \times R \mid f(\bar{\boldsymbol{\sigma}}, \kappa) \le 0 \right\}$$

$$\dot{\kappa} = \left\| \dot{\boldsymbol{\varepsilon}}^p \right\| \qquad f(\bar{\boldsymbol{\sigma}}, \kappa) = \sqrt{\bar{\boldsymbol{\sigma}} : \mathbf{F} : \bar{\boldsymbol{\sigma}}} - \sigma_y(\kappa)$$

$$\sigma_y(\kappa) = 1 + \sigma_H \left(1 - e^{-\kappa/\kappa_h}\right) \qquad \omega(\kappa) = \omega_c \left(1 - e^{-\kappa/\kappa_s}\right)$$





Princip maxima plastické disipace

$$D^p\left(\boldsymbol{\tau}, \dot{\boldsymbol{\varepsilon}}^p\right) = \boldsymbol{\tau} : \dot{\boldsymbol{\varepsilon}}^p$$

$$D^p\left(\overline{\boldsymbol{\sigma}}, \dot{\boldsymbol{\varepsilon}}^p\right) = \max_{\tau \in E_\sigma} D^p\left(\boldsymbol{\tau}, \dot{\boldsymbol{\varepsilon}}^p\right)$$

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \frac{\partial f}{\partial \overline{\boldsymbol{\sigma}}}$$

$$\dot{\lambda} \geq 0 \qquad f(\overline{\boldsymbol{\sigma}}, \kappa) \leq 0 \qquad \dot{\lambda} f(\overline{\boldsymbol{\sigma}}, \kappa) = 0$$

$$f(\overline{\boldsymbol{\sigma}}, \kappa) = \sqrt{\overline{\boldsymbol{\sigma}} : \mathbf{F} : \overline{\boldsymbol{\sigma}}} - \sigma_y(\kappa)$$ konvexní

ρ ... 1-porozita

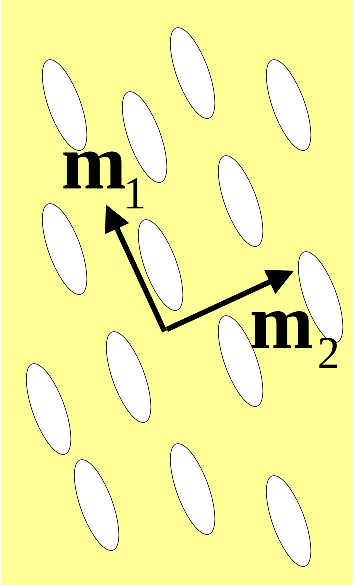
$$\mathbf{M} = \sum_{I=1}^{3} m_I \, \mathbf{m}_I \otimes \mathbf{m}_I$$ ... fabric tensor

Hlavní hodnoty

$$\sum_{I=1}^{3} m_I = 3$$

Hlavní směry

$$\mathbf{m}_I \cdot \mathbf{m}_J = \delta_{IJ}$$



$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p \right)$$

$$\mathbf{C}_e = \mathbf{D}_e^{-1}$$

C^e_{1111}	C^e_{1122}	C^e_{1133}			
C^e_{2211}	C^e_{2222}	C^e_{2233}			
C^e_{3311}	C^e_{3322}	C^e_{3333}			
			$4C^e_{2323}$		
				$4C^e_{3131}$	
					$4C^e_{1212}$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p\right)$$

$$\mathbf{C}_e = \mathbf{D}_e^{-1}$$

$$C^e_{IIII} = \frac{1}{E_0}$$

$$C^e_{IIJJ} = -\frac{\nu_0}{E_0}$$

$$C^e_{IJIJ} = \frac{1}{4G_0}$$

C^e_{1111}	C^e_{1122}	C^e_{1133}			
C^e_{2211}	C^e_{2222}	C^e_{2233}			
C^e_{3311}	C^e_{3322}	C^e_{3333}			
			$4C^e_{2323}$		
				$4C^e_{3131}$	
					$4C^e_{1212}$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p \right)$$

$$\mathbf{C}_e = \mathbf{D}_e^{-1}$$

$$C^e_{IIII} = \frac{1}{E_0 \rho^k}$$

$$C^e_{IIJJ} = -\frac{\nu_0}{E_0 \rho^k} \qquad \left(I \neq J \right)$$

$$C^e_{IJIJ} = \frac{1}{4 G_0 \rho^k} \qquad \left(I \neq J \right)$$

C^e_{1111}	C^e_{1122}	C^e_{1133}			
C^e_{2211}	C^e_{2222}	C^e_{2233}			
C^e_{3311}	C^e_{3322}	C^e_{3333}			
			$4C^e_{2323}$		
				$4C^e_{3131}$	
					$4C^e_{1212}$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p \right)$$

$$\mathbf{C}_e = \mathbf{D}_e^{-1}$$

$$C_{IIII}^e = \frac{1}{E_0 \rho^k m_I^{2l}}$$

$$C_{IIJJ}^e = -\frac{\nu_0}{E_0 \rho^k m_I^l m_J^l} \quad \left(I \neq J \right)$$

$$C_{IJIJ}^e = \frac{1}{4G_0 \rho^k m_I^l m_J^l} \quad \left(I \neq J \right)$$

C_{1111}^e	C_{1122}^e	C_{1133}^e			
C_{2211}^e	C_{2222}^e	C_{2233}^e			
C_{3311}^e	C_{3322}^e	C_{3333}^e			
			$4C_{2323}^e$		
				$4C_{3131}^e$	
					$4C_{1212}^e$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_{\mathrm{e}} : \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}}\right)$$

$$\sqrt{\bar{\boldsymbol{\sigma}} : \mathbf{F} : \bar{\boldsymbol{\sigma}}} - \sigma_{\mathrm{y}}(\kappa)$$

$$\mathbf{C}_e = \mathbf{D}_e^{-1}$$

$$\mathbf{F}$$

Tenzor poddajnosti

Tenzor v podmínce plasticity

$$C_{IIII}^{e} = \frac{1}{E_0 \rho^k m_I^{2l}}$$

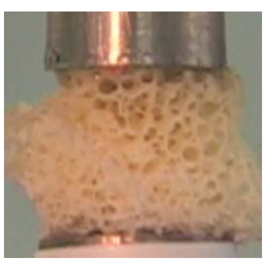
$$C_{IIJJ}^{e} = -\frac{\nu_0}{E_0 \rho^k m_I^l m_J^l}$$

$$C_{IJIJ}^{e} = \frac{1}{4 G_0 \rho^k m_I^l m_J^l}$$

$$F_{IIII} = \frac{1}{\sigma_0^2 \rho^{2p} m_I^{4q}}$$

$$F_{IIJJ} = -\frac{\chi_0}{\sigma_0^2 \rho^{2p} m_I^{2q} m_J^{2q}}$$

$$F_{IJIJ} = \frac{1}{4 \tau_0^2 \rho^{2p} m_I^{2q} m_J^{2q}}$$



DÁNO $\boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\varepsilon}^{p}_{n}, \kappa_n$	$\bar{\boldsymbol{\sigma}}^{tr} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}^{p}_{n}\right)$ $f^{tr} \equiv f\left(\bar{\boldsymbol{\sigma}}^{tr}, \kappa_n\right) = \sqrt{\bar{\boldsymbol{\sigma}}^{tr} : \mathbf{F} : \bar{\boldsymbol{\sigma}}^{tr}} - \sigma_y(\kappa_n)$
IF	$f^{tr} \leq 0$ $\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}^{tr}, \boldsymbol{\varepsilon}^{p}_{n+1} = \boldsymbol{\varepsilon}^{p}_{n}, \omega_{n+1} = \omega_n$
ELSE	$\left.\begin{aligned} &\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}^{tr} - \Delta\kappa \frac{\mathbf{D}_e : \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}}{\left\Vert \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1} \right\Vert} \\ &\sqrt{\bar{\boldsymbol{\sigma}}_{n+1} : \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}} - \sigma_y(\kappa_n + \Delta\kappa) = 0 \end{aligned}\right\} \bar{\boldsymbol{\sigma}}_{n+1}, \Delta\kappa$
	$\boldsymbol{\varepsilon}^{p}_{n+1} = \boldsymbol{\varepsilon}^{p}_{n} + \Delta\kappa \frac{\mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}}{\left\Vert \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1} \right\Vert}$ $\kappa_{n+1} = \kappa_n + \Delta\kappa$ $\omega_{n+1} = \omega(\kappa_{n+1})$ $\bar{\boldsymbol{\sigma}}_{n+1} = \left(1 - \omega_{n+1}\right)\bar{\boldsymbol{\sigma}}_{n+1}$

DÁNO $\boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\varepsilon}^p_n, \kappa_n$	$\bar{\boldsymbol{\sigma}}^{tr} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}^p_n\right)$ $f^{tr} \equiv f\left(\bar{\boldsymbol{\sigma}}^{tr}, \kappa_n\right) = \sqrt{\bar{\boldsymbol{\sigma}}^{tr} : \mathbf{F} : \bar{\boldsymbol{\sigma}}^{tr}} - \sigma_y(\kappa_n)$
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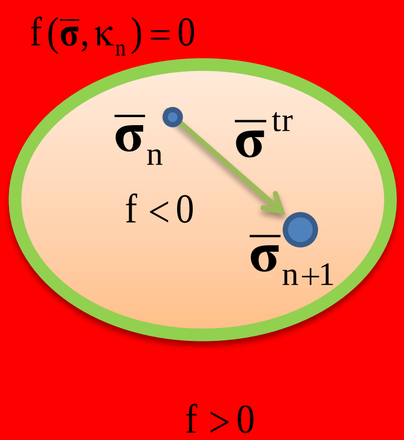
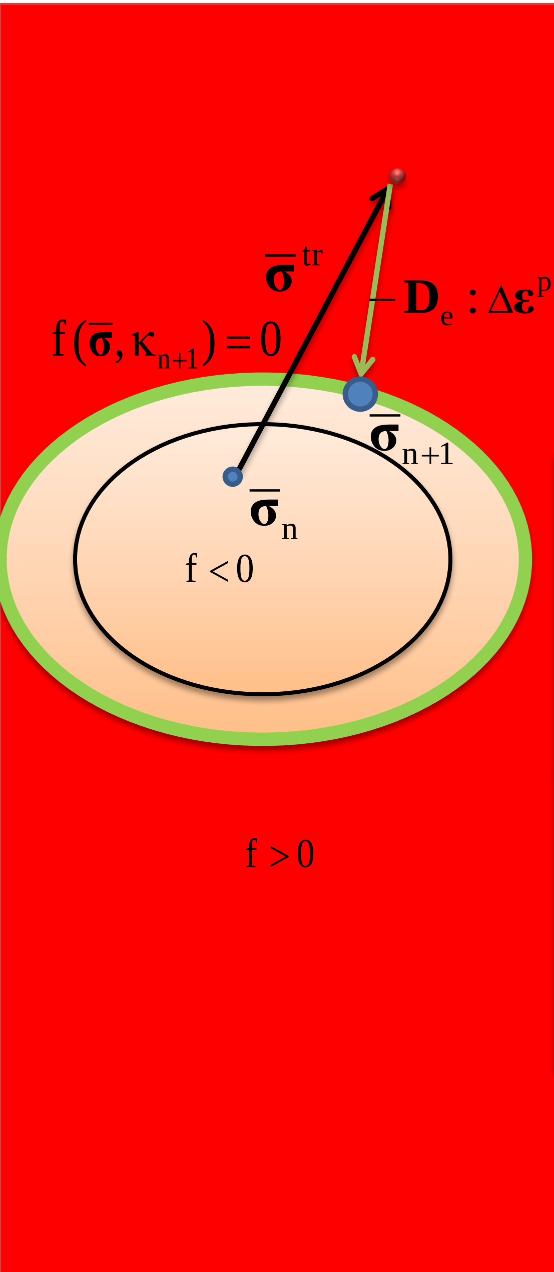
	$\boldsymbol{\varepsilon}^p_{n+1} = \boldsymbol{\varepsilon}^p_n + \Delta\kappa \dfrac{\mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}}{\left\Vert \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1} \right\Vert}$ $\kappa_{n+1} = \kappa_n + \Delta\kappa$ $\omega_{n+1} = \omega(\kappa_{n+1})$ $\boldsymbol{\sigma}_{n+1} = \left(1 - \omega_{n+1}\right)\bar{\boldsymbol{\sigma}}_{n+1}$



DÁNO	$\boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\varepsilon}^p_n, \kappa_n$	$\bar{\boldsymbol{\sigma}}^{tr} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}^p_n\right)$ $f^{tr} \equiv f\left(\bar{\boldsymbol{\sigma}}^{tr}, \kappa_n\right) = \sqrt{\bar{\boldsymbol{\sigma}}^{tr} : \mathbf{F} : \bar{\boldsymbol{\sigma}}^{tr}} - \sigma_y(\kappa_n)$
IF	$f^{tr} \leq 0$	
	$\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}^{tr}, \boldsymbol{\varepsilon}^p_{n+1} = \boldsymbol{\varepsilon}^p_n, \omega_{n+1} = \omega_n$	
ELSE	$\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}^{tr} - \Delta\kappa \dfrac{\mathbf{D}_e : \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}}{\left\lVert \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1} \right\rVert}$ $\sqrt{\bar{\boldsymbol{\sigma}}_{n+1} : \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}} - \sigma_y(\kappa_n + \Delta\kappa) = 0$	$\Rightarrow \bar{\boldsymbol{\sigma}}_{n+1}, \Delta\kappa$
	$\boldsymbol{\varepsilon}^p_{n+1} = \boldsymbol{\varepsilon}^p_n + \Delta\kappa \dfrac{\mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1}}{\left\lVert \mathbf{F} : \bar{\boldsymbol{\sigma}}_{n+1} \right\rVert}$ $\kappa_{n+1} = \kappa_n + \Delta\kappa$ $\omega_{n+1} = \omega(\kappa_{n+1})$ $\bar{\boldsymbol{\sigma}}_{n+1} = \left(1 - \omega_{n+1}\right)\bar{\boldsymbol{\sigma}}_{n+1}$	



DÁNO $\boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\varepsilon}_n^p, \kappa_n$	$\overline{\boldsymbol{\sigma}}^{tr} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n^p\right)$ $f^{tr} \equiv f\left(\overline{\boldsymbol{\sigma}}^{tr}, \kappa_n\right) = \sqrt{\overline{\boldsymbol{\sigma}}^{tr} : \mathbf{F} : \overline{\boldsymbol{\sigma}}^{tr}} - \sigma_y(\kappa_n)$
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ELSE	$\overline{\boldsymbol{\sigma}}_{n+1} = \overline{\boldsymbol{\sigma}}^{tr} - \Delta\kappa \dfrac{\mathbf{D}_e : \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}}{\left\lVert \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1} \right\rVert}$ $\sqrt{\overline{\boldsymbol{\sigma}}_{n+1} : \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}} - \sigma_y(\kappa_n + \Delta\kappa) = 0$ $\Rightarrow \overline{\boldsymbol{\sigma}}_{n+1}, \Delta\kappa$
	$\boldsymbol{\varepsilon}_{n+1}^p = \boldsymbol{\varepsilon}_n^p + \Delta\kappa \dfrac{\mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}}{\left\lVert \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1} \right\rVert}$ $\kappa_{n+1} = \kappa_n + \Delta\kappa$
	$\omega_{n+1} = \omega(\kappa_{n+1})$ $\boldsymbol{\sigma}_{n+1} = (1 - \omega_{n+1})\overline{\boldsymbol{\sigma}}_{n+1}$

Linearizace algoritmu návratu na plochu plasticity

$$\mathbf{S} = \left[C_e + \Delta\kappa \, \mathbf{N} \right]^{-1}$$

$$\delta\bar{\boldsymbol{\sigma}}^{(n+1)} = \underbrace{\left(\mathbf{S} - \frac{\mathbf{S} : \bar{\mathbf{n}} \otimes \bar{\mathbf{n}} : \mathbf{S}}{\bar{\mathbf{n}} : \mathbf{S} : \bar{\mathbf{n}} + \sigma_Y' / \|\mathbf{f}\|} \right)}_{\boldsymbol{\Theta}^{(n+1)}} : \delta\boldsymbol{\varepsilon}^{(n+1)}$$

$$\delta\kappa^{(n+1)} = \underbrace{\frac{\bar{\mathbf{n}} : \mathbf{S}}{\bar{\mathbf{n}} : \mathbf{S} : \bar{\mathbf{n}} + \sigma_Y' / \|\mathbf{f}\|}}_{\boldsymbol{\eta}^{(n+1)}} : \delta\boldsymbol{\varepsilon}^{(n+1)}$$

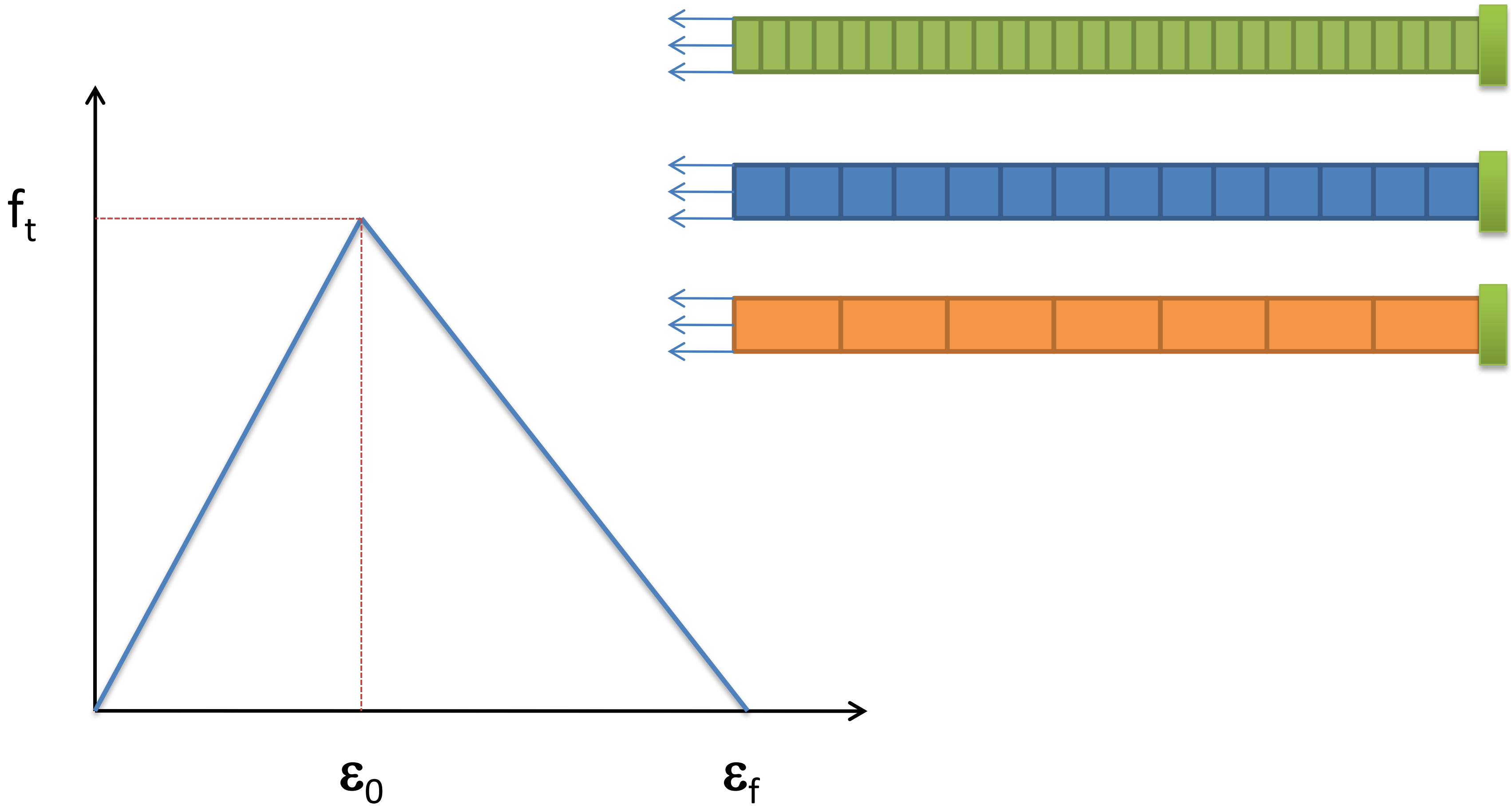
$$\delta\boldsymbol{\sigma}^{(n+1)} = \left[\left(1 - \omega^{(n+1)}\right) \boldsymbol{\Theta}^{(n+1)} - \omega' \bar{\boldsymbol{\sigma}}^{(n+1)} \otimes \boldsymbol{\eta}^{(n+1)} \right] : \delta\boldsymbol{\varepsilon}^{(n+1)}$$

Linearizace algoritmu návratu na plochu plasticity

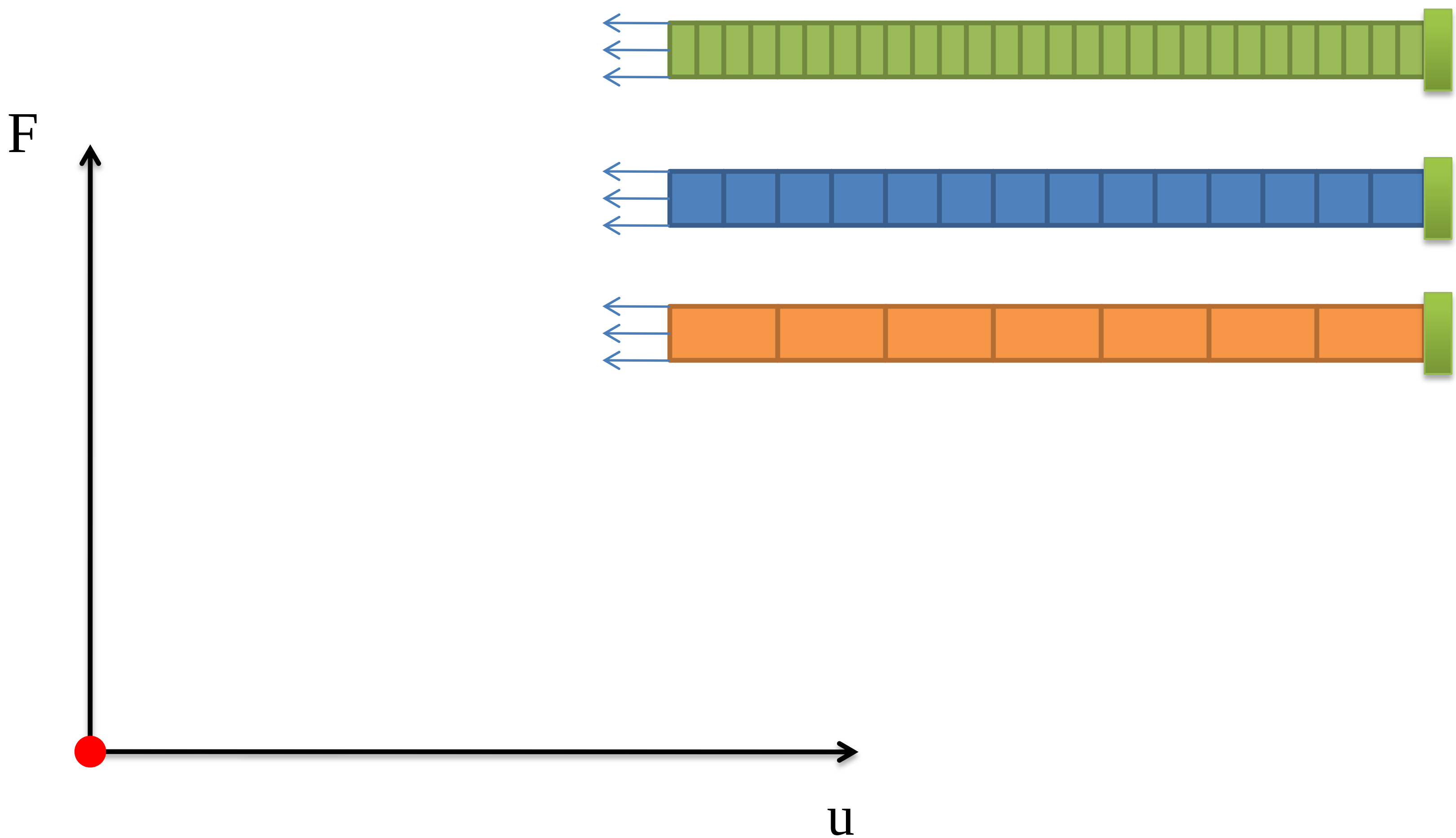
$$\mathbf{D}_{\mathrm{pd}} = \left[\left(1 - \omega^{(n+1)}\right) \mathbf{\Theta}^{(n+1)} - \omega' \overline{\boldsymbol{\sigma}}^{(n+1)} \otimes \boldsymbol{\eta}^{(n+1)} \right]$$

$$\mathbf{K}^{\mathrm{e}} = \int_{\mathrm{V}} \mathbf{B}^{\mathrm{T}} \mathbf{D}_{\mathrm{pd}} \mathbf{B} \, \mathrm{dV}$$

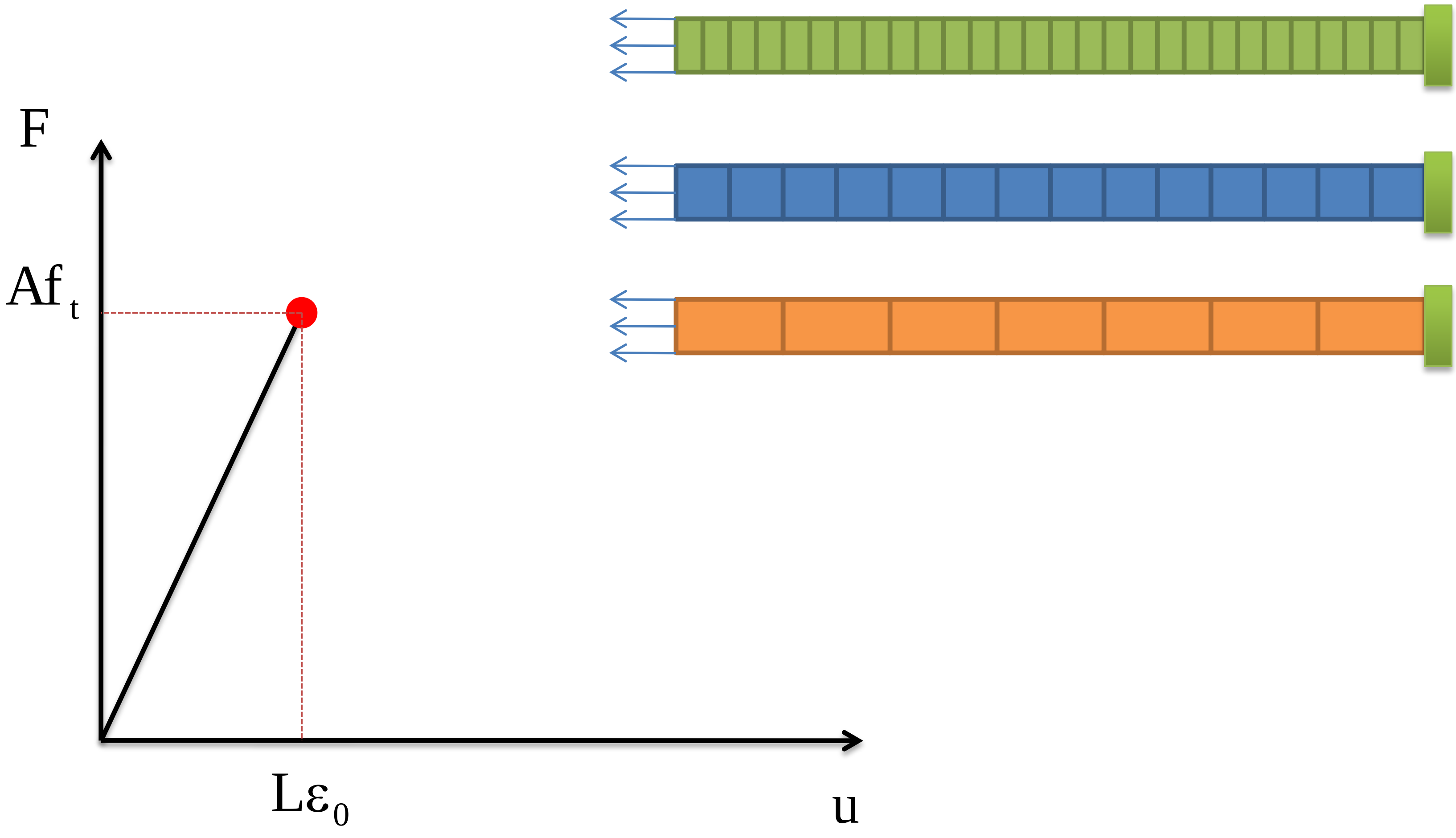
Lokalizace nepružných vlastností



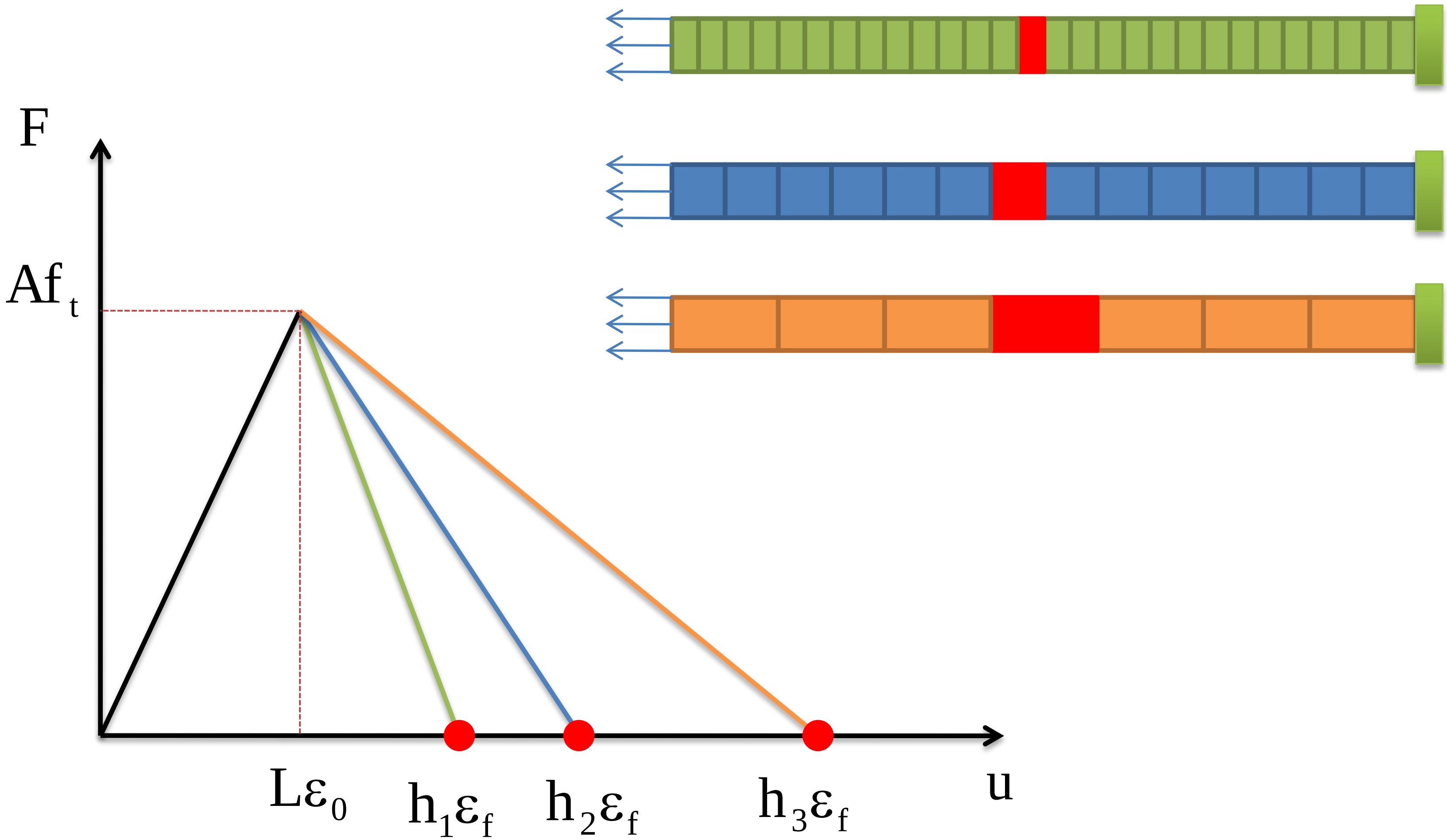
Lokalizace nepružných vlastností



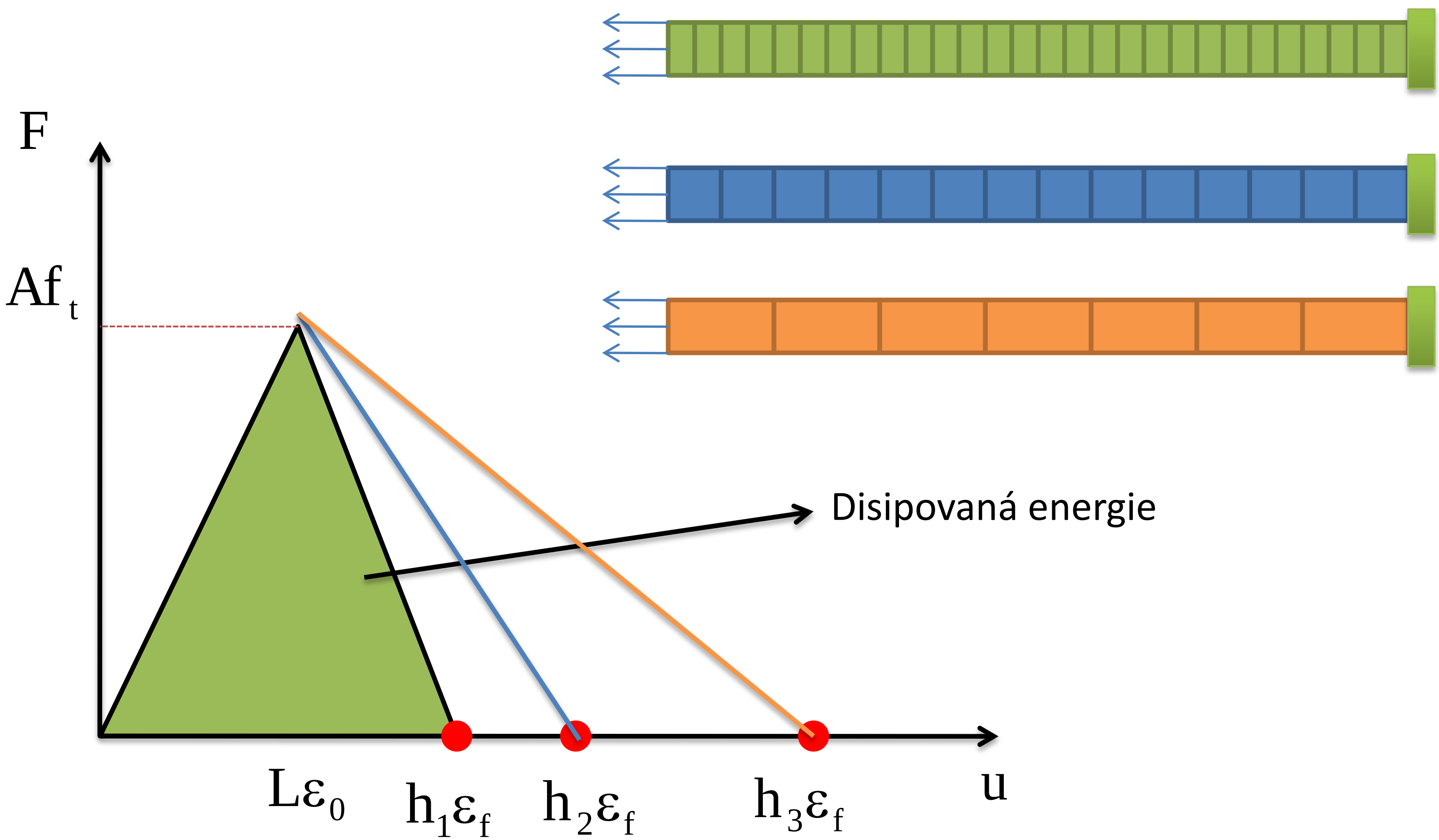
Lokalizace nepružných vlastností



Lokalizace nepružných vlastností



Lokalizace nepružných vlastností



Lokální modely

Silná(rank-1) elipticita

Symetrické tenzory

$$\mathbf{m} \otimes \mathbf{n} : \mathbf{D}_{\mathrm{pd}} : \mathbf{m} \otimes \mathbf{n} \geq \alpha \|\mathbf{n}\|^2 \|\mathbf{m}\|^2$$

$$\alpha(x) \geq 0 \qquad \mathbf{m}, \mathbf{n} \in \mathrm{R}^3$$

$$\det\left(\mathbf{n} \cdot \mathbf{D}^{s}{}_{\mathrm{pd}} \cdot \mathbf{n}\right) \geq 0$$

Nesymetrické tenzory

elipticita

$$\det\left(\mathbf{n} \cdot \mathbf{D}_{\mathrm{pd}} \cdot \mathbf{n}\right) \geq 0$$

$$\mathbf{n} \in \mathrm{R}^3$$

Lokální modely

$$\mathbf{m} \otimes \mathbf{n} : \mathbf{D}_{\mathrm{pd}} : \mathbf{m} \otimes \mathbf{n} \geq \alpha \|\mathbf{n}\|^2 \|\mathbf{m}\|^2$$

$$\det\left(\mathbf{n} \cdot \mathbf{D}_{\mathrm{pd}} \cdot \mathbf{n}\right) \geq 0$$

- Neobjektivní popis po ztrátě eliptičnosti
  - Fyzikální význam : Nulová disipovaná energie
  - Numerický význam : Patologická závislost na síti

Regularizace

- Gradientní modely
  - explicitní $\bar{\kappa} = \kappa + c\nabla^2\kappa$
  - Implicitní $\bar{\kappa} - c\nabla^2\bar{\kappa} = \kappa$ (Helmholzova rovnice)
- Nelokální modely

$$\bar{\kappa}(x) = \int_V \alpha(x,\xi)\kappa(\xi)dV$$

- Časově závislé modely(viskoplasticita)

Regularizace

- Gradientní modely
  - explicitní $\bar{\kappa} = \kappa + c\nabla^2\kappa$
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- Nelokální modely

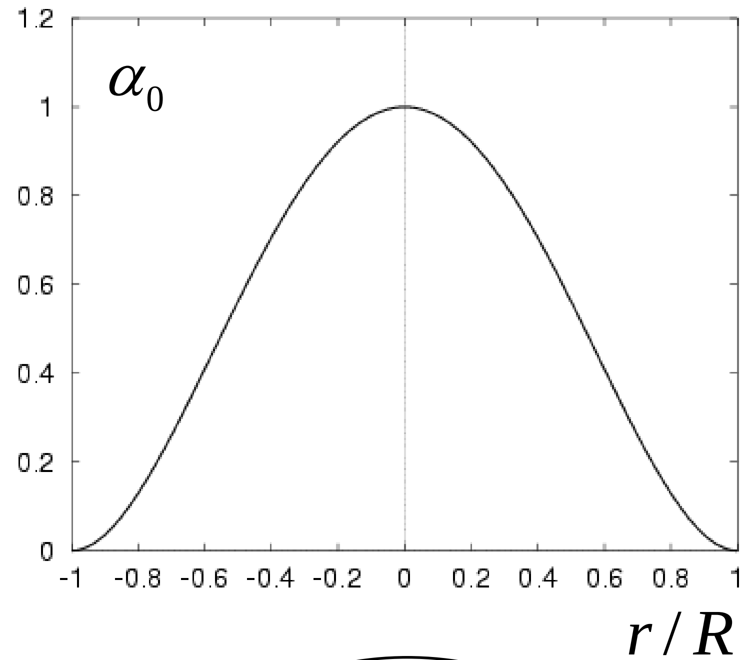
$$\bar{\kappa}(x) = \int_V \alpha(x,\xi)\kappa(\xi)dV$$

- Časově závislé modely(viskoplasticita)

Nelokální model

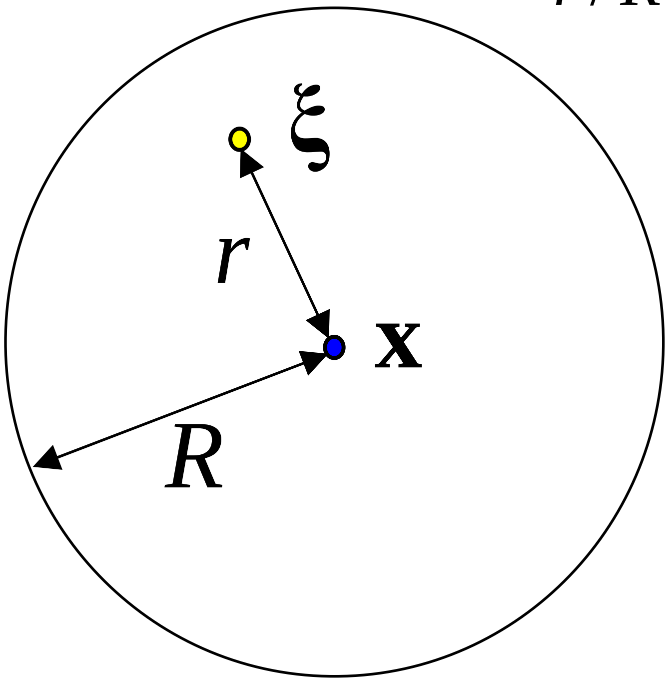
Nelokální kumulovaná pl. deformace:

$$\bar{\kappa}(\mathbf{x}) = \int_V \alpha(\mathbf{x},\boldsymbol{\xi})\kappa(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}$$



$$\alpha_0(r) = \left\langle 1 - \frac{r^2}{R^2} \right\rangle^2$$

$$\alpha(\mathbf{x},\boldsymbol{\xi}) = \frac{\alpha_0(\|\mathbf{x}-\boldsymbol{\xi}\|)}{\int_V \alpha_0(\|\mathbf{x}-\boldsymbol{\eta}\|)\,\mathrm{d}\boldsymbol{\eta}}$$

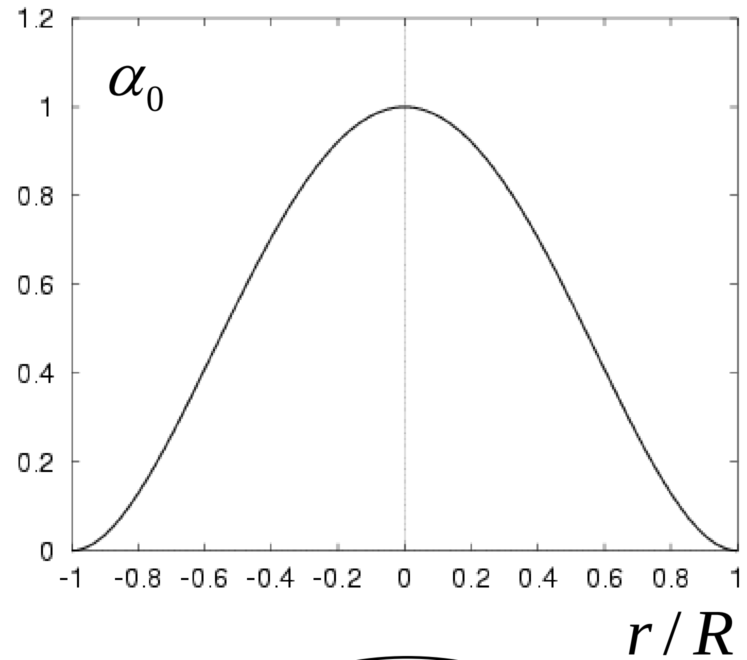


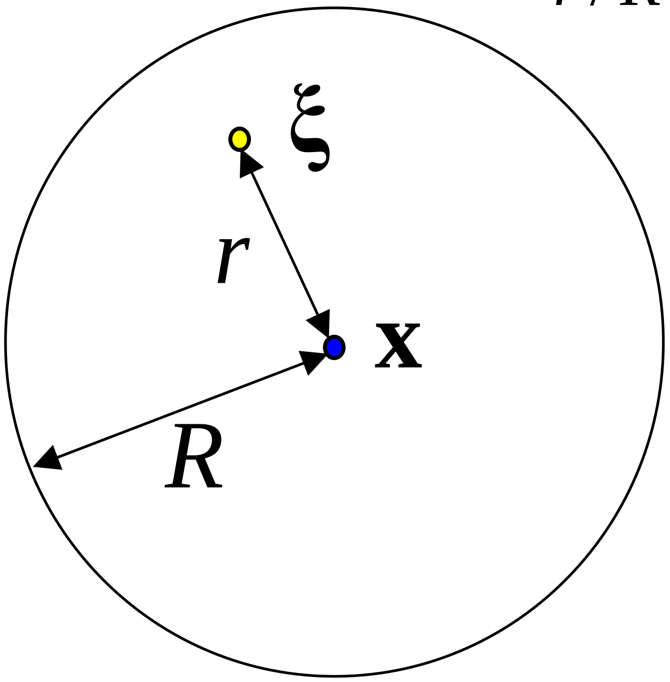
Nelokální model

poškození

$$\omega = \omega(\hat{\kappa})$$

$$\hat{\kappa}(\mathbf{x}) = m\bar{\kappa}(\mathbf{x}) + (1-m)\kappa(\mathbf{x})$$





DÁNO $\boldsymbol{\varepsilon}_{n+1}, \boldsymbol{\varepsilon}_n^p, \kappa_n$	$\overline{\boldsymbol{\sigma}}^{tr} = \mathbf{D}_e : \left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n^p\right)$ $f^{tr} \equiv f\left(\overline{\boldsymbol{\sigma}}^{tr}, \kappa_n\right) = \sqrt{\overline{\boldsymbol{\sigma}}^{tr} : \mathbf{F} : \overline{\boldsymbol{\sigma}}^{tr}} - \sigma_y(\kappa_n)$
IF	$f^{tr} \leq 0$ $\overline{\boldsymbol{\sigma}}_{n+1} = \overline{\boldsymbol{\sigma}}^{tr}, \boldsymbol{\varepsilon}_{n+1}^p = \boldsymbol{\varepsilon}_n^p, \omega_{n+1} = \omega_n$
ELSE	$\left.\begin{aligned} &\overline{\boldsymbol{\sigma}}_{n+1} = \overline{\boldsymbol{\sigma}}^{tr} - \Delta\kappa \frac{\mathbf{D}_e : \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}}{\left\Vert \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1} \right\Vert} \\ &\sqrt{\overline{\boldsymbol{\sigma}}_{n+1} : \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}} - \sigma_y(\kappa_n + \Delta\kappa) = 0 \end{aligned}\right\} \overline{\boldsymbol{\sigma}}_{n+1}, \Delta\kappa$
	$\boldsymbol{\varepsilon}_{n+1}^p = \boldsymbol{\varepsilon}_n^p + \Delta\kappa \frac{\mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1}}{\left\Vert \mathbf{F} : \overline{\boldsymbol{\sigma}}_{n+1} \right\Vert}$ $\kappa_{n+1} = \kappa_n + \Delta\kappa \longrightarrow \hat{\kappa}_{n+1} = (1-m)\kappa_{n+1} + m\overline{\kappa}_{n+1}$
	$\omega_{n+1} = \omega(\hat{\kappa}_{n+1})$ $\boldsymbol{\sigma}_{n+1} = \left(1 - \omega_{n+1}\right)\overline{\boldsymbol{\sigma}}_{n+1}$

Vektor vnitřních sil:

$$\mathbf{f}_{\text{int}} = \int_V \mathbf{B}^T \boldsymbol{\sigma}\, \mathrm{d}\mathbf{x} \approx \sum_r w_r \mathbf{B}_r^T \boldsymbol{\sigma}_r$$

Suma přes Gaussovy body

Napětí v Gaussově bodě r : $\boldsymbol{\sigma}_r = (1-\omega_r)\bar{\boldsymbol{\sigma}}_r$

return mapping

Poškození v GP r : $\omega_r = \omega(\hat{\kappa}_r) =$ $\bar{\boldsymbol{\sigma}}_r = \boldsymbol{\theta}_r(\boldsymbol{\varepsilon}_r)$

$$= \omega\left(m\bar{\kappa}_r + (1-m)\kappa_r\right)$$

nelokální κ v GP r :

$$\bar{\kappa}_r = \int_V \alpha(\mathbf{x}_r, \boldsymbol{\xi})\kappa(\boldsymbol{\xi})d\boldsymbol{\xi} \approx$$

$$\approx \sum_s \alpha_{rs}\kappa_s$$

Deformace GP r : $\boldsymbol{\varepsilon}_r = \mathbf{B}_r\mathbf{d}$

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Deformace GP r : $\boldsymbol{\varepsilon}_r = \mathbf{B}_r \mathbf{d}$

$$\approx \sum_s \alpha_{rs} \kappa_s$$

$\kappa_s = \eta_s(\boldsymbol{\varepsilon}_s)$ $\boldsymbol{\varepsilon}_s = \mathbf{B}_s \mathbf{d}$

Linearizace:

$$\delta \mathbf{f}_{\text{int}} = \sum_r w_r \mathbf{B}_r^T \delta \boldsymbol{\sigma}_r$$

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$$= \omega\left(m \bar{\kappa}_r + (1-m)\kappa_r\right)$$

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$$\delta \bar{\boldsymbol{\sigma}}_r = \boldsymbol{\Theta}_r \delta \boldsymbol{\varepsilon}_r$$

Poškození v GP r: $\omega_r = \omega(\hat{\kappa}_r) =$

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Deformace GP $\delta \boldsymbol{\varepsilon}_r = \mathbf{B}_r \delta \mathbf{d}$

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$$\delta \omega_r = \omega_r' \left[m\,\delta \bar{\kappa}_r + (1-m)\,\delta \kappa_r \right]$$

nelokální κ v GP r :

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Deformace GP r : $\delta \boldsymbol{\varepsilon}_r = \mathbf{B}_r \delta \mathbf{d}$

$$\kappa_s = \eta_s(\boldsymbol{\varepsilon}_s) \qquad \boldsymbol{\varepsilon}_s = \mathbf{B}_s \mathbf{d}$$

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v GP r:

$$\delta\bar{\kappa}_r = \sum_s \alpha_{rs}\delta\kappa_s$$

Deformace GP r: $\delta\boldsymbol{\varepsilon}_r = \mathbf{B}_r\delta\mathbf{d}$

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nelokální κ
v GP r :

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r : $\delta\boldsymbol{\varepsilon}_r = \mathbf{B}_r \delta\mathbf{d}$

$$\delta\kappa_r = \boldsymbol{\eta}_r^T \delta\boldsymbol{\varepsilon}_r$$

$$\delta\kappa_s = \boldsymbol{\eta}_s^T \delta\boldsymbol{\varepsilon}_s$$

$$\delta\boldsymbol{\varepsilon}_s = \mathbf{B}_s \delta\mathbf{d}$$

Nelokální tečná tuhost

$$\delta \mathbf{f}_{\text{int}} = \mathbf{K}\,\delta \mathbf{d}$$

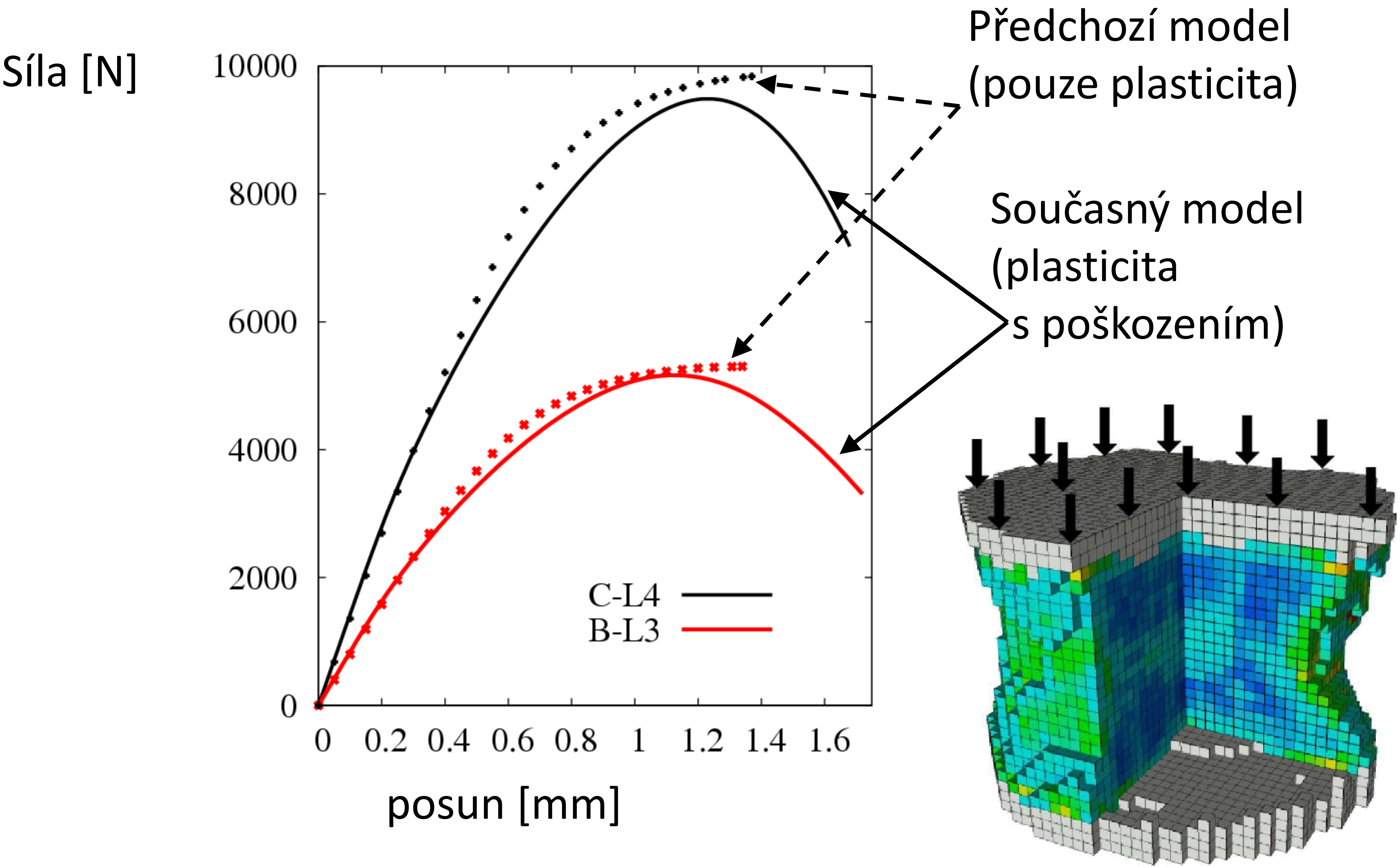
$$\mathbf{K} = \sum_r w_r \left(1 - \omega_r\right) \mathbf{B}_r^T \boldsymbol{\Theta}_r \mathbf{B}_r$$

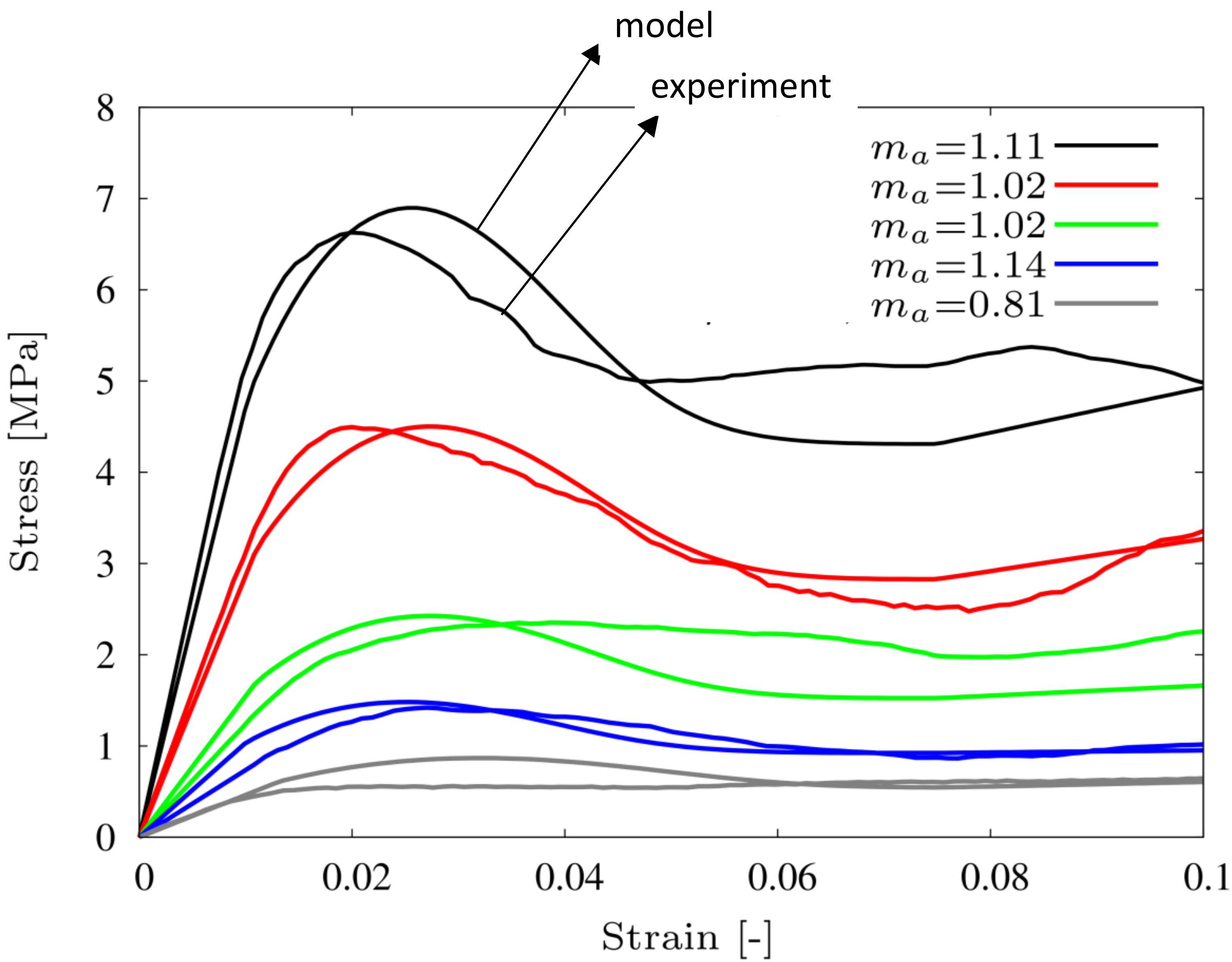
Nelokální tečná tuhost

$$\delta \mathbf{f}_{\text{int}} = \mathbf{K}\,\delta \mathbf{d}$$

$$\mathbf{K} = \sum_r w_r \left(1 - \omega_r\right) \mathbf{B}_r^T \boldsymbol{\Theta}_r \mathbf{B}_r - \left(1 - m\right) \sum_r w_r \omega_r' \mathbf{B}_r^T \boldsymbol{\sigma}_r \boldsymbol{\eta}_r^T \mathbf{B}_r$$

$$- m \sum_r \sum_s w_r \omega_r' \alpha_{rs} \mathbf{B}_r^T \boldsymbol{\sigma}_r \boldsymbol{\eta}_s^T \mathbf{B}_s$$

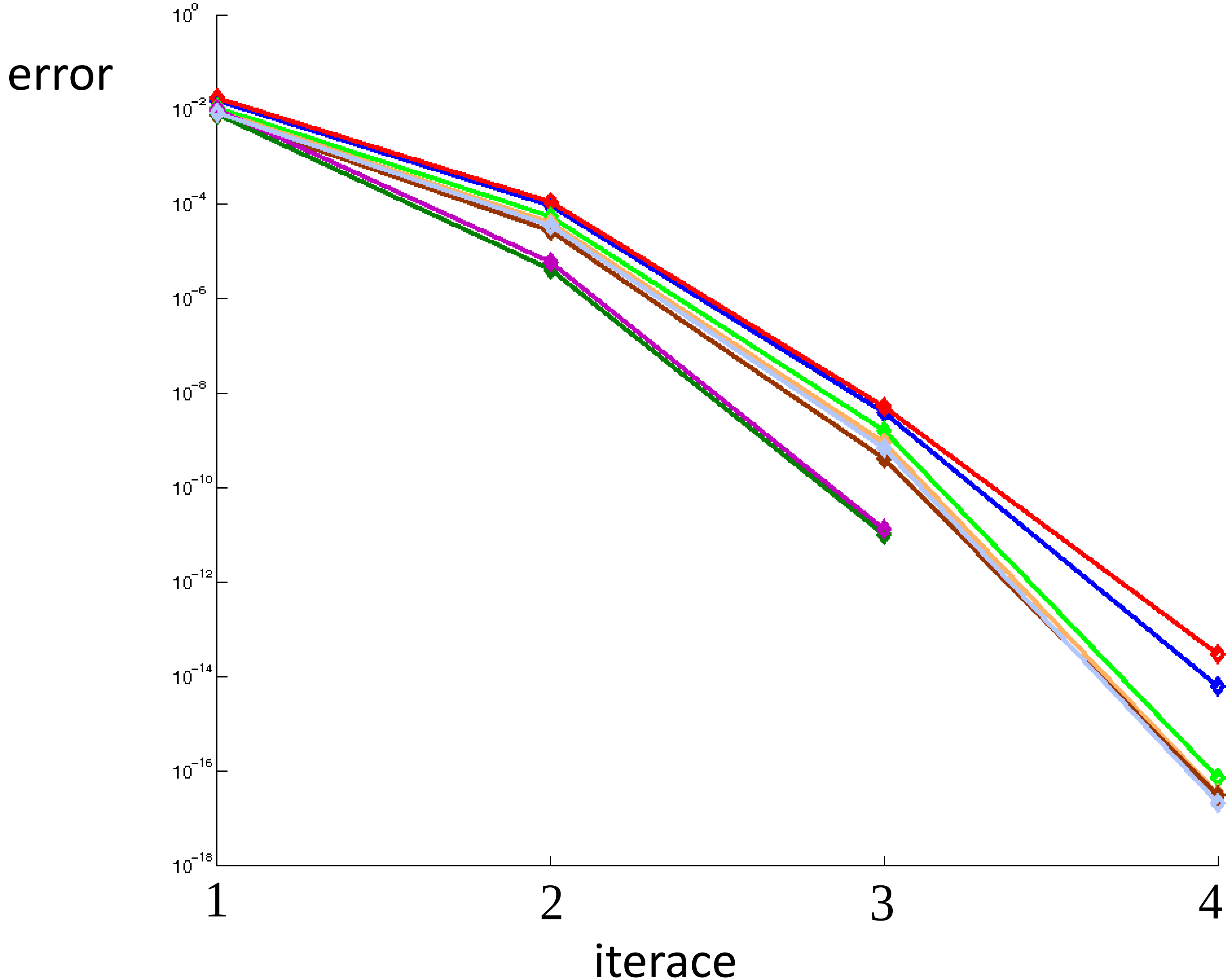

Předchozí model
(pouze plasticita)
Současný model
(plasticita
s poškozením)
Síla [N]
10000
8000
6000
4000
2000
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
posun [mm]
C-L4
B-L3



model
experiment
m_a=1.11
m_a=1.02
m_a=1.02
m_a=1.14
m_a=0.81
Stress [MPa]
Strain [-]
0
1
2
3
4
5
6
7
8
0
0.02
0.04
0.06
0.08
0.1

κ

ω


error
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
10^-16
10^-18
1
2
3
4
iterace

Počet nenulových prvků matice tuhosti