A new reconstruction-enhanced discontinuous Galerkin method for time-dependent PDEs

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In this work a new numerical method for time-dependent PDEs is introduced, numerically tested and the first steps toward an error analysis are undertaken. In the finite volume method, which uses piecewise constant approximate solutions, higher order approximations are obtained by a reconstruction procedure. This constructs a higher order piecewise polynomial solution from the piecewise constant data on appropriate neighborhoods (so-called stencils) of individual elements. The disadvantage of this procedure is that for higher orders, the reconstruction stencil for each element is prohibitively large and the method is very memory and time consuming. On the other hand, the discontinuous Galerkin (DG) method handles piecewise polynomials of arbitrary order in a natural way. We propose to incorporate a similar reconstruction procedure into DG schemes. This allows us to substantially increase the accuracy of the underlying scheme, while only a minimal reconstruction stencil is used. For example, a DG solution of order N on simpicial meshes allows us to reconstruct a solution of order $2N+1$ in 2D and $3N+2$ in 1D, using only the von Neumann neighborhood of each element as a stencil. The accuracy and effectiveness of the new method is demonstrated on a convection-diffusion problem. Approximation properties of the reconstruction operator and the relation to standard DG schemes are analyzed.